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BERTRAND COMPETITION FOR INPUTS, FORWARD CONTRACTS, AND WALRASIAN OUTCOMES

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by

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ABSTRACT

A model of market-making merchants who buy from suppliers and sell to consumers is treated as a two-stage Bertrand game. Winner-take-all competition for the inputs distinguishes these models from fixed or flexible capacity models. When stocks are acquired in the first stage, followed by sales to consumers, a Subgame Perfect Nash Equilibrium (SPNE) exists if and only if the elasticity of demand at the Walrasian price is at least unity; in that case, the unique SPNE is Walrasian. On the other hand, when forward contracts are sold to consumers in the first stage, followed by acquisition of stocks, a unique SPNE always exists and is Walrasian.

1. INTRODUCTION.

Consider a model of merchants who obtain stock from suppliers and resell it to consumers. Suppose the merchants face Bertrand price competition on both sides. Such a model of "Bertrand merchants" is a possible way to endogenize the role of the Walrasian auctioneer in a pure exchange economy. Indeed, a typical story of how markets reach equilibrium involves the notion of arbitraging middle men. But before such a story can be given a rigorous foundation, we must have a sensible solution to the Bertrand pricing game.

One of the troublesome aspects of Bertrand price competition with capacity constraints is the absence of a pure-strategy Nash Equilibrium (NE). Only recently has it been shown that even a mixed-strategy NE exists in general [Dasgupta and Maskin (1982)]. While the discontinuous behavior of consumers (everyone going to the seller with the lowest price no matter how small the price difference) may be somewhat unrealistic, such behavior is not the cause of the lack of a pure-strategy NE. Consumer behavior can be smoothed and still no pure-strategy NE will emerge (unless there is sufficient smoothing to make the profit functions concave). It is the natural non-concavity of the Bertrand seller's profit function that rules out pure-strategy NE. Simply assuming away this non-concavity is not a satisfactory solution.

There are two interesting special cases in which Bertrand competition does have a pure-strategy NE. The first case is when total capacity is at least twice the quantity demanded at the Walrasian price. Then the Walrasian price is the NE strategy for every seller. The second case is when total capacity does not exceed the monopoly quantity. Then the market clearing price (which is the capacity-constrained monopoly price) is the NE strategy for every player. These two cases suggest that bringing in the choice of capacity might alter the outcome. Indeed , Kreps and Scheinkmann (1983) show that in a two-stage game of capacity choice followed by Bertrand price competition, the unique perfect NE is the Cournot outcome.

Since the acquisition of stock is analogous to capacity choice, one might reasonably hope that the NE of a model of "Bertrand merchants" would have a sensible outcome, in contrast to the fixed capacity case. This paper investigates such a model.

First, consider a two-stage game in which stocks are acquired in the first stage and sold in the second stage. The supply function and the demand function are non-stochastic and common knowledge to all merchants. Each merchant sets (1) a bid price for the first stage and (2) an ask price for the second stage conditional on everyone's stocks. Sufficient regularity is assumed to ensure a unique Walrasian price (i.e.

the common bid and ask price that equates supply and demand) and a unique "sales revenue maximizing price" (i.e. the price a monopolist would charge in the second stage with no capacity constraints). Assume that initial inventories of stock are zero for all merchants. For example, merchants upon waking up in the morning go to the farmers and obtain fresh produce, and then travel to the city to sell this fresh produce to urban consumers. Assuming that the produce perishes overnight, the merchants' inventories would be zero every morning.[1] Also assume that there are no binding capacity constraints on the amount of stock any merchant can acquire during the first stage. The highest bidder acquires all the stock. In the event of equal bids, the supplies are allocated equally among all merchants. In the second stage, the consumer sales are rationed "uniformly" (see section 2).

I will show that there exists a Sugame Perfect NE (SPNE) of this two-stage game if <u>and only if</u> the Walrasian price is at least as great as the sales revenue maximizing price (equivalently, if and only if the elasticity of demand at the Walrasian price is at least unity). When a SPNE exists, the SPNE bid and ask prices are the Walrasian price, and this <u>Walrasian outcome is the unique SPNE outcome</u>. (In this case, the SPNE price also happens to be the price which drives all

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[1]This dynamic story is told solely for the purpose of motivating the assumptions of the one-shot two-stage game. The repeated version of the game will be briefly addressed at the end of the paper.

merchant profits to zero.) This result holds for any number of merchants greater than one.

Heuristically, the reason for the non-existence case is that (i) an outcome with positive monopoly profits cannot be an equilibrium because the merchants would out bid each other, each believing he would be a second stage monopolist; (ii) in the limit of such a bidding war where expected monopoly profits are zero, the merchants share the second stage and get negative expected profits, so the limit bids are not a SPNE either. Analysis of a discrete version of the game, in which bid strategies are limited to a finite partition of the price space with finer and finer partitions, suggests that when the Walrasian price is less than the sales revenue maximizing price (elasticity less than unity) consumers will generally be at the mercy of a monopolist.

The non-existence result disappears under a particular first-stage allocation scheme. In the case of equal bids, instead of equal division of the stocks suppose all stocks are allocated to only one merchant chosen randomly. Then the "zero monopoly profits" outcome (the limit described above) is the unique SPNE outcome. That is, prices are bid up to the point of driving all monopoly profits to zero, and consumers face the randomly chosen monopolist who charges the sales revenue maximizing price.

The second model considered is like the first except that the stages are reversed. Merchants sell forward contracts to consumers for delivery, and then go to the suppliers to obtain the needed stocks. A penalty for default is imposed sufficient to prevent defaults. <u>The unique SPNE outcome of this two stage game is Walrasian</u>; i.e. the bid and ask prices equal the Walrasian price. The sales revenue maximizing price (and elasticity of demand) no longer plays a crucial role. This result holds for any number of merchants greater than one.

The Walrasian outcome in the forward contracting model when the elasticity of demand is less than unity stands in sharp contrast to the former model without forward contracts: both (i) non-existence when bidding ties are resolved by equal allocation, and (ii) the zero monopoly profits outcome when bidding ties are resolved randomly. This contrast makes it tempting to suggest that one might find more forward contracting in markets for which the elasticity of demand at the Walrasian price is less than unity since consumers would have an incentive to change the game.

These results are also dramatically different from the results of the fixed capacity models and the capacity choice models [Benoit and Krishna (1985); Kreps and Scheinkman (1983); Vives (1983)]. In those models the outcome is either monopolistic or Cournot, not Walrasian. The <u>important modelling difference</u> is that the latter models involve <u>no competition for</u>

the inputs, while the Bertrand merchant model has "winner-take-all" competition for the inputs. The lesson is that the input side of the market can have significant effects on the nature of the output equilibrium. Assuming unrestricted availability of inputs at a constant marginal cost may generate singular results. The different outcomes indicate also an incentive for vertical integration.

The paper is organized as follows. Section 2 sets out the notation and basic assumptions; it also shows that uniform rationing is essentially the same as proportional rationing with resale by consumers. Section 3 analyzes the case of a single monopolist in order to provide a benchmark for comparison. Section 4 develops the model without forward contracts, and section 5 develops the model with forward contracts. The results are discussed in section 6. Several involved proofs are gathered in an Appendix.

2. NOTATION and RATIONING SCHEMES.

Let (p_{bi}, p_{ai}) denote the bid and ask prices respectively of the i^{th} merchant. These prices are the strategy variables of the merchants. Given a bid price p_{bi}, the merchant acquires whatever stock the suppliers are willing to provide at that price. Let x_i denote the stock on hand of the i^{th} merchant after the first stage. Given p_{ai}, the merchant sells whatever

stock the consumers are willing to buy (up to x_1).

Let S(p) denote the supply function of the suppliers and let D(p) denote the demand function of the consumers. Assume that both are continuously differentiable with $S' > 0 > D'$. The requirement that the supply function be upward sloping and not vertical will be relaxed in sections 4 and 5 after the main results are presented.

Assume that there is a unique Walrasian equilibrium price p_c such that $D(p_c) = S(p_c)$. Further assume that the sales revenue function pD(p) is strictly concave, so there is a unique sales revenue maximizing price $\hat{p}$.

The ask price subgame with quantity constraints must specify a rationing scheme that determines which consumers are able to purchase from the lowest priced merchant, for example from merchant 2 when $p_{a1} > p_{a2}$. The <u>uniform rationing scheme</u> would take the stock of merchant 2 and distribute it uniformly among all consumers. If D(p,y) is the representative demand function with fixed exogenous income y, then the residual demand facing merchant 1 is $D[p_{a1},(p_{a1}-p_{a2})x_2+y] - x_2$. Letting $\beta \equiv \partial D/\partial y$, the first-order approximation of residual demand is

$$D(p_{a1},y) - x_2 + \beta(p_{a1}-p_{a2})x_2 \ . \tag{1}$$

The <u>proportional rationing scheme</u> would take a randomly

chosen representative fraction α of the consumers and satisfy their demand completely; $\alpha = \min\{x_2/D(p_{a2}),1\}$. Residual demand for merchant 1 is $(1-\alpha)D(p_{a1})$ provided that resale by the lucky consumers is not permitted. In the absence of such prohibition, the lucky consumers would be willing to resell the quantity $x_2 - \alpha D[p_{a1},(p_{a1}-p_{a2})x_2/\alpha + y]$. Then, the residual demand facing merchant 1 would be

$$(1-\alpha)D(p_{a1},y) - x_2 + \alpha D[p_{a1},(p_{a1}-p_{a2})x_2/\alpha + y]$$

$$\approx D(p_{a1},y) - x_2 + \beta(p_{a1}-p_{a2})x_2 \;, \qquad (2)$$

Note that the first-order approximations of residual demand, (1) and (2), are identical. In other words, proportional rationing with resale is essentially equivalent to uniform rationing.[2] As is often done [e.g. Kreps and Scheinkman], we will assume that β is negligible.

3. BENCHMARK MONOPOLY OUTCOME.

As a point of reference it is useful to know how a monopolistic merchant would set prices. Given a bid price p_b,

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[2]Proportional rationing without resale was studied by Beckman (1965) and recently by Allen and Hellwig (1984). Davidson and Deneckere (1982) discuss the implications of this scheme in a capacity choice game similar to Kreps and Scheinkman. The NE of our merchant game under this rationing scheme remains an open question.

stocks at the end of the first stage would be $x = S(p_b)$. Then, in the second stage, sales would be $\min\{D(p_a),x\}$. Hence, total profits from both stages are

$$\pi(p_b,p_a) = p_a\min\{D(p_a),x\} - p_bS(p_b) \ . \tag{3}$$

It is straightforward to show that the monopolist will always want to clear the market exactly, which implies that the price strategies are confined to the set such that $D(p_a) = S(p_b)$. By the assumptions on supply and demand and the implicit function theorem, there is a continuously differentiable function $g(\cdot)$ such that $D[g(p_b)] = S(p_b)$, and $g'(p_b) = S'(p_b)/D'(g) < 0$. This locus is shown in Figure 1 where it is drawn as a fairly straight line.

The optimum bid price is that which maximizes $\pi[p_b,g(p_b)] = [g(p_b) - p_b]S(p_b)$. The first-order conditions are

$$(1 - g')S(p_b) = [g(p_b) - p_b]S'(p_b) \ , \tag{4}$$

which is the price version of marginal revenue equals marginal cost. Let p_{bm} denote the solution, and $p_{am} \equiv g(p_{bm})$. Figure 1 depicts these for the case in which a unique solution exists. It is noteworthy that p_{am} is higher and p_{bm} is lower than the sales revenue maximizing price $\hat{p}$. This follows because marginal cost is positive for the former and zero (by definition) for the latter. It is also noteworthy that p_{am} is

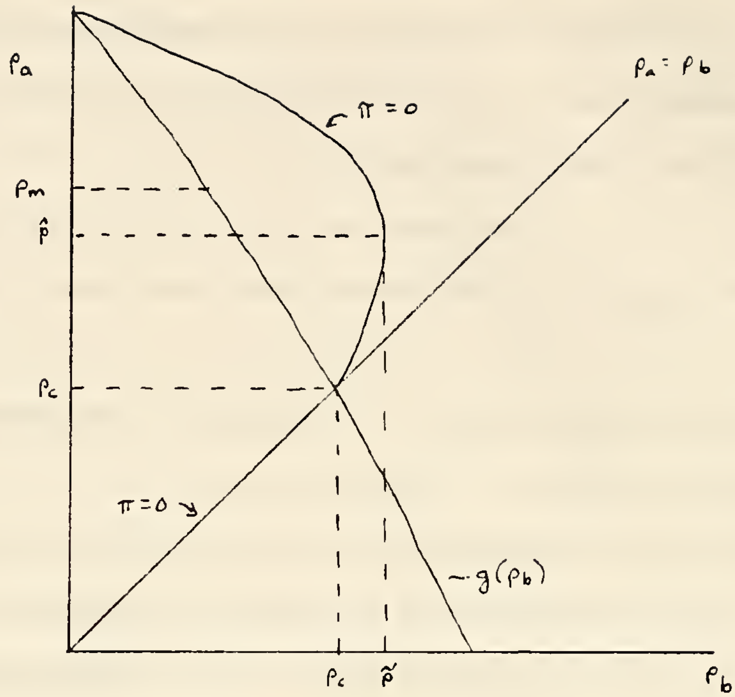

Figure 1.

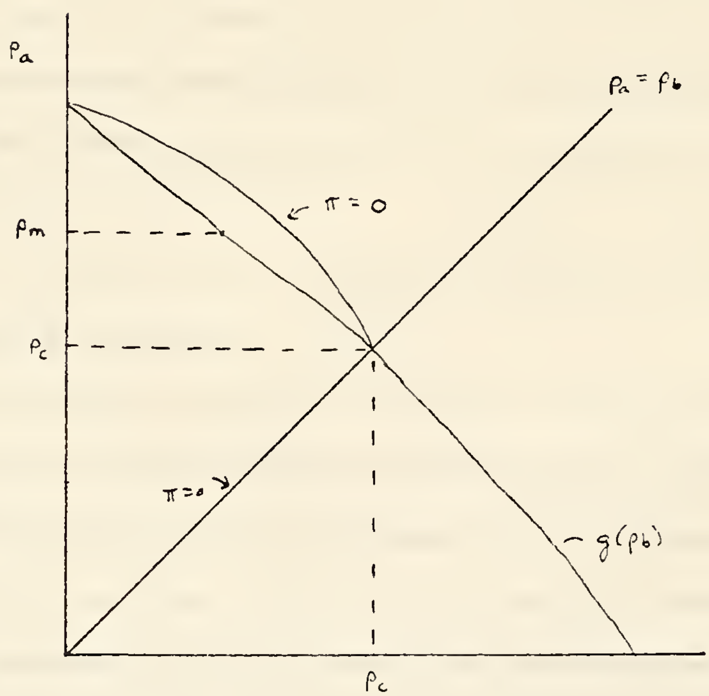

Figure 2.

higher and p_{bm} is lower than the Walrasian price p_c. This follows because the right-hand-side of eq(4) is zero when $p_b = p_c$, while the left-hand-side is positive.

4. DUOPOLIST MERCHANTS WITHOUT FORWARD CONTRACTS.

Let us first consider the case of two merchants and later generalize to $N \geq 2$ merchants. The solution is derived by backwards induction. Given stocks (x_1, x_2), the merchants face Bertrand price competition for consumers in the second stage. For any ask prices (p_{a1}, p_{a2}), let $z_i(p_{a1}, p_{a2})$ be the resulting sales for merchant i.

$$z_1(p_{a1}, p_{a2}) = \begin{cases} \min\{x_1, D(p_{a1})\}, & \text{if } p_{a1} < p_{a2} \\ \min\{x_1, \max[D(p_{a1})/2, D(p_{a1}) - x_2]\}, & \text{if } p_{a1} = p_{a2} \\ \min\{x_1, \max[D(p_{a1}) - x_2, 0]\}, & \text{if } p_{a1} > p_{a2}. \end{cases}$$

Similarly for the second merchant. We assume here "uniform" rationing and negligible income effects, which (as seen in section 2) is equivalent to proportional rationing with consumer resale. This subgame has been thoroughly analyzed by Kreps and Scheinkman (1983). In particular, they prove that there is a unique (equilibrium) expected revenue function.

Let $ER_i(x_1,x_2)$ be this expected revenue from a second stage NE. Note that $ER_1(0,x_2) = 0$, for any x_2. In other words, zero stocks always yields zero second stage revenue. Further, $ER_2(0,x_2) = M(x_2)$, where $M(x) \equiv \max\{pD(p) \mid D(p) \leq x\}$ is the maximum monopoly revenues attainable with stock x. In other words, if some merchant has zero stocks, then the other merchant is a monopolist in the second stage.

The first stage acquisitions, given pure strategies (p_{b1},p_{b2}) and Bertrand competition are denoted by $x_1(p_{b1},p_{b2})$.

$$x_1(p_{b1},p_{b2}) = \begin{cases} S(p_{b1}) , & \text{if } p_{b1} > p_{b2} \\ S(p_{b1})/2 , & \text{if } p_{b1} = p_{b2} \\ 0 , & \text{if } p_{b1} < p_{b2}. \end{cases}$$

The outcome is "winner take all" (except in the event of equal bids) where "all" entails not only the first stage acquisitions but also the prize of being a second stage monopolist. In the event of equal bids, the merchants share the supplies equally.[3] The alternative of breaking a bidding tie by choosing a winner randomly will be considered later.

Let F_i denote a cumulative probability distribution which

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[3]The results are robust to allocations "nearly" equal in the event of bidding ties.

characterizes merchant i's first stage mixed strategy. These mixed strategies may include a countable number of atoms (probability mass concentrated on a point). Let $\{p_k\}$ denote the collection of prices for which some merchant has an atom, and let F_{ik} denote the probability mass at p_k. It is also notationally convenient to let S_k denote $S(p_k)$, and $\mu(p) \equiv M[S(p)] - pS(p)$, the net profits to the winner bidding p (assuming no ties). Because of the possibility of atoms, the cumulative distributions are not necessarily continuous from below. Let $F_i(p^-)$ denote the limit as prices approach p from below; that is, the probability that $p_{bi} < p$. Obviously, $F_i(p^-) = F_i(p)$ if and only if there is no atom at p.

Given this notation, the total expected profits from (F_1,F_2) are given by

$$E\pi_1(F_1,F_2) = \int [\mu(p_{b1})F_2(p_{b1}^-)]dF_1 \quad + \tag{5}$$
$$\Sigma_k\{ER_1[S_k/2,S_k/2] - p_kS_k/2\}F_{1k}F_{2k} \ .$$

For the analysis, it is convenient to define $\bar{p}$ such that $\mu(\bar{p}) = 0$; i.e. the highest bid price that will yield non-negative profits for the winner. Clearly, the support of the optimal F_i cannot include any prices greater than $\bar{p}$.

<u>Lemma 1</u>. A necessary condition for F_i to be a SPNE strategy is that all mass be concentrated at $\bar{p}$.

PROOF: (1) Suppose both merchants have an atom at $p_k < \acute{p}$. Merchant 1 can do better by moving this atom up to $p_k+\varepsilon$ but still less than $\acute{p}$ and less than the next atom (p_{k+1}) if there is an atom at a higher price. This claim is established in two steps. First, from Kreps and Scheinkman (1983), it can be shown that $ER_1(x,x) \leq M(2x)/2$ [see Lemma A of Appendix]. Second, using this fact and eq(5) note that for small ε the gains from such a move are on the order of $\mu(p_k)$ while the losses are on the order of half that. Therefore, there can be no matched atoms below $\acute{p}$ in a SPNE.

(2) Next consider the necessary requirements for F_1 on $[0,\acute{p})$. Since F_1 must be a best response to F_2, the integrand of the first line of eq(5) must be independent of p_{b1} on the support of F_1, which implies

$$F_2(p_{b1}^-) \quad \propto \quad 1/\mu(p_{b1}) \quad . \tag{6}$$

The further requirement that F_2 be non-decreasing puts a lower bound on the support at least as great as the monopoly price of section 3. An equation analogous to (6) must hold for F_1. Let $\underline{p}_1$ denote the lower bound of the support of F_1. Suppose $\underline{p}_1 = \underline{p}_2$. Then eq(6) requires F_1 to have an atom at $\underline{p}_1$, contradicting step (1) above. Suppose $\underline{p}_2 < \underline{p}_1 < \acute{p}$. But note that merchant 2 can do better by putting all the probability mass lying below $\underline{p}_1$ at $\underline{p}_1+\delta$ for some $\delta > 0$. We can pick δ so there is no atom at $\underline{p}_1+\delta$, hence zero probability of a tie.

Moving mass from below to above $\underline{p}_1$ (but still below $\hat{p}$) gives merchant 2 a positive chance of winning and gaining positive monopoly profits (since $\underline{p}_2 < \hat{p}$). In other words, the assumption that $\underline{p}_2 < \underline{p}_1 < \hat{p}$ leads to a contradiction. Assuming that $\max\{\underline{p}_i\} < \hat{p}$, the only remaining alternative is $\underline{p}_1 < \underline{p}_2 < \hat{p}$. But with the same arguments we can derive a symmetric contradiction. Therefore, we must have $\underline{p}_i = \hat{p}$ for both merchants. The conclusion is that a SPNE strategy must be degenerate with all mass at $\hat{p}$. Q.E.D.

From eq(5), the expected profits from this pure strategy are $E\pi = ER[S(\hat{p})/2, S(\hat{p})/2] - \hat{p}S(\hat{p})/2$, dropping subscripts because of symmetry. Recalling that $ER(x,x) \leq M(2x)/2$, we have $E\pi \leq \{M[S(\hat{p})]-\hat{p}S(\hat{p})\}/2 = \mu(\hat{p})/2 \equiv 0$. Clearly, if this $E\pi < 0$, then no SPNE exists because the merchants can simply bid zero and secure a return of 0.

<u>Proposition 1</u>. If $p_c < \hat{p}$, then no SPNE exists.[4]

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[4]While I believe that SPNE are the most relevant and interesting, it is noteworthy that Lemma 1 and Proposition 1 apply to all NE. To see this recall that the difference between a SPNE and a non-SPNE is that the latter involves strategies off the equilibrium path that are not NE of the subgames and which act as threats affecting the equilibrium outcome. Intuitively, however, there are no effective threats in this merchant game. The only way one merchant can affect the other in the second stage is to have some stock, which entails a bidding tie. But any such threat is nullified by a higher bid by the threatened party unless both are already bidding $\hat{p}$. But the latter case is moot because neither would want to bid $\hat{p}$ in the first place. A formal proof could be provided.

PROOF: First recall that sales revenue for the winner of the first stage is $p_a \min\{D(p_a), x\}$, so for $x \geq D(\hat{p})$, the maximum second stage revenue is $\hat{p}D(\hat{p})$. Given $p_c < \hat{p}$, if p_c were the winning bid, then $x = S(p_c) > D(\hat{p})$, which implies that $M(x) = \hat{p}D(\hat{p})$, which is strictly greater than $p_c D(p_c)$. Hence, $\mu(p_c) = M(x) - p_c S(p_c) > p_c[D(p_c) - S(p_c)] = 0$, so $\hat{p}$ must be greater than p_c. Moreover, the ask price associated with bid $\hat{p}$ is $\hat{p}$. In other words, if the merchants bid $\hat{p}$, then $E\pi \leq 0$, with equality iff both ask $\hat{p}$ in the second stage. However, the Bertrand NE of the second stage will have both merchants putting probability mass at prices below $\hat{p}$. To see this, note that by undercutting, a merchant can nearly double his revenues; hence, both playing pure strategies $\hat{p}$ cannot be a second stage NE, and no merchant will put positive probability on any $p > \hat{p}$. Therefore, $E\pi < 0$, which completes the proof.

Q.E.D.

Figure 1 depicts the locus of $E\pi = 0$, with $\hat{p}$ occurring at the right-most point on this locus above the 45° line. All points between this locus and the p_a axis and above the 45° line yield positive profits to the winner.

The reason for non-existence is that the sum of the payoff functions, $E\pi_1 + E\pi_2$, is not upper semi-continuous, thus not satisfying the conditions for Theorem 5 of Dasgupta and Maskin [in particular, see their Example 3]. This can be illustrated by looking at a discrete version of the game in which the

strategies are limited to a finite partition of the price space, making the partitions finer and finer with the continuum as the limit. The mixed-strategy equilibria of the discrete versions converge to a pure-strategy of asking the sales revenue maximizing price and bidding high enough to drive expected profits to zero. In the sequence, the merchants are second-stage monopolists with probability approaching one, but in the limit they are in a symmetric N-player second stage Bertrand game with negative expected revenues. In the original game, these pure strategies do not constitute a SPNE because each merchant can ensure himself of non-negative profits by bidding zero. This analysis suggests that when the Walrasian price is less than the sales revenue maximizing price, consumers will generally be at the mercy of a monopolist.

<u>Proposition 2</u>. If $p_c \geq \hat{p}$, there exists a unique SPNE. The corresponding strategies are (i) to bid $p_{b1} = p_c$, and (ii) given first stage stock acquisitions, to play second stage NE strategies. In equilibrium, $p_{a1} = p_c$ (= $\hat{p}$) for both merchants.

PROOF: Given $p_{b1} = p_c$ for both merchants, the unique NE of the second stage is $p_{a1} = p_c$ yielding $E\pi = 0$, as anticipated. To see this notice that undercutting p_c will yield losses since p_c was the cost, and pricing over p_c will also yield losses because $\hat{p}$ is the sale revenue maximizing price. Then bidding and asking p_c

is a SPNE. Next note that $\hat{p} = p_c$, and hence, no other bids could be SPNE strategies. (Any bid yielding positive expected profits will be defeated by a slightly higher bid until all expected profits are driven out.) Thus, bidding and asking p_c is the unique SPNE outcome. Q.E.D.

Figure 2 depicts the case of Proposition 2. The $E\pi = 0$ locus is non-positively sloped at (p_c, p_c), which graphically shows why $\hat{p} = p_c$.

It should be noted that $\hat{p} \leq p_c$ does not imply that the monopoly price of section 3 is less than or equal to p_c. Indeed, as noted in section 3, the opposite is generally true, regardless of the relation of $\hat{p}$ and p_c. This fact is also shown in Figure 2.

Under the assumption that $pD(p)$ is strictly concave, we can restate Propositions 1 and 2 in terms of the elasticity of demand evaluated at p_c. Let η_c denote this elasticity (expressed as a non-negative number). Then, $\hat{p} \leq p_c$ iff $\eta_c \geq 1$. In other words, a SPNE exists only for markets with sufficiently high elasticity of demand at the Walrasian price. Otherwise, a SPNE does not exist, and the analysis of the discrete version on a partitioned price space with finer and finer partitions suggests that either consumers will almost surely face a sequence of monopolists, <u>or</u> perhaps intermediation by merchants will simply not occur.

Propositions 1 and 2 hold for any number of Bertrand merchants greater than one. The proofs mimic those given above; details are in the Appendix.

Consider an alternative first-stage allocation scheme that does not ensure equal stocks for both merchants in the event of equal bids. In particular, suppose one of the merchants is chosen at random, say with probability $\alpha \in (0,1)$. This winner gets all the stock and is a monopolist in the second stage. The reader can verify that Lemma 1 continues to hold, so the only possible SPNE strategy is to bid $\breve{p}$. However, now the second stage outcome in the event both merchants bid $\breve{p}$ is not a Bertrand game, but instead is a monopoly with revenue $M[S(\breve{p})]$. Expected profits are $E\pi = \alpha\mu(\breve{p}) = 0$, as anticipated by the merchants. Therefore, $\breve{p}$ is a SPNE outcome even when $p_c < \hat{p}$. Since $\breve{p} = p_c$ when $p_c \geq \hat{p}$, the unique SPNE outcome has bids of $\breve{p}$ followed by the monopolist ask price (constrained by stocks $S(\breve{p})$). This is reminiscent of Demsetz's (1968) conjecture that competition for a monopoly license will drive rents to zero.

The specifications of section 2 rule out the cases of vertical and horizontal supply. The case of vertical supply goes through provided that $D(0)$ exceeds the fixed supply, which ensures that $p_c > 0$. This condition avoids the discontinuity in supply at $p_b = 0$. When this condition is not fulfilled, we essentially have a horizontal supply at zero marginal costs.

The case of horizontal supply defines a distinctly different game, since the bidding subgame no longer has the winner-take-all characteristic. In particular, we get the Kreps and Scheinkman "capacity choice" game in which there is no competition for inputs and the outcome is Cournot. Thus, unrestricted availability of inputs at a constant marginal cost is a singular case.[5]

5. DUOPOLIST MERCHANTS WITH FORWARD CONTRACTS.

Reversing the stages of the previous game results in a model of merchants who first sell forward contracts to consumers and then obtain supplies to fill these contracts. Bertrand price competition is assumed in both stages. In addition, the penalty for default is assumed to be sufficiently severe to prevent any default in equilibrium.[6]

Obviously, the monopolist solution to this forward contract model is exactly the same as described in section 3 (with the order reversed). The duopolist solution, however, is different from section 4.

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[5]An intermediate result could probably be obtained in a model in which each merchant had exclusive access to some supplies while both could buy also in a common market.

[6]A penalty that gives a defaulting merchant a non-positive payoff will suffice to deter default. This assumption also rules out threats off the equilibrium path involving default.

The first stage is now the consumer market. Since there are no capacity constraints in forward contracts, the lowest ask price gets all the orders; there is no need for a rationing mechanism when ask prices differ. In the event of equal ask prices, assume (for now) that orders are split equally.[7] Let $y_1(p_{a1},p_{a2})$ denote the realized orders in stage one.

$$y_1(p_{a1},p_{a2}) = \begin{cases} D(p_{a1}) & , \text{ if } p_{a1} < p_{a2} \\ D(p_{a1})/2 & , \text{ if } p_{a1} = p_{a2} \\ 0 & , \text{ if } p_{a1} > p_{a2} . \end{cases}$$

As in section 4, let $x_1(\, , \,)$ denote the realized stock, now from stage two. The no-default condition requires that $y_1(p_{a1},p_{a2}) \leqq x_1(p_{b1},p_{b2})$. Conditional on the first stage play, no-default puts the following requirement on second stage bids of merchant 1 (and symmetrically for merchant 2):

(i) If $p_{a1} < p_{a2}$, then either (a) $p_{b1} \geqq S^{-1}[D(p_{a1})]$ and $p_{b1} > p_{b2}$, or (b) $p_{b1} = S^{-1}[2D(p_{a1})] = p_{b2}$.

(ii) If $p_{a1} = p_{a2}$, then $p_{b1} \geqq \max\{S^{-1}[D(p_{a1}),p_{b2}\}$.

(iii) If $p_{a1} > p_{a2}$, then p_{b1} is unrestricted.

$S^{-1}(x)$ is the bid price that will result in supply x in a

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[7]Both proportional and uniform rationing would have equal sharing in this case since both have equal potential capacity.

monopoly setting. It exists and is unique whenever $S' > 0$ as assumed.

Condition (i) says that when a merchant wins the first stage subgame, then he must bid sufficiently high to secure the necessary supplies to fill his orders. Condition (ii) says that when both merchants share the orders, each must bid sufficiently high to fill his orders. Condition (iii) is superfluous, since if a merchant loses the first stage subgame, he would not want any supplies. Subgame perfection requires that $p_{b1} < p_{b2}$. For the same reason, (i)(b) is also superfluous.

Define strategy σ_1 as follows: (1) $p_{a1} = p_c$; and (2) $p_{b1} = S^{-1}[D(p_{a1})]$ if $p_{a1} \leq p_{aj}$, and $p_{b1} \in [0, S^{-1}[D(p_{aj})])$ otherwise. To illucidate (2), note that when $p_{a1} < p_{a2}$ merchant 1 gets all the orders and bids $S^{-1}[D(p_{a1})]$; then merchant 2 can bid anything from 0 up to but not including $S^{-1}[D(p_{a1})]$ all of which give him no supplies. This leeway in the losing bid implies a multiplicity of equilibrium strategies; but this is the only non-uniqueness. The <u>Walrasian outcome</u> is defined as $p_{ai} = p_c = p_{bi}$ for all i.

<u>Proposition 3</u>. The family of symmetric strategies $\{\sigma_1, \sigma_2\}$ coincides with the set of SPNE strategies, all of which yield the Walrasian outcome.

PROOF: (1) σ constitutes a NE. The payoffs are $E\pi = 0$. If $p_{a1} > p_c$, merchant 1 obtains no orders and under bids, yielding a zero payoff, which is no better. If $p_{a1} < p_c$, then merchant 1 gets all the orders, and must bid at least $S^{-1}[D(p_{a1})]$ which is greater than p_c; hence, $E\pi < 0$. (2) σ is subgame perfect because the specified p_{bi} are NE of the second stage subgame. If $p_{a1} < p_{a2}$, then merchant 1 is the winner and he must bid at least $S^{-1}[D(p_{a1})]$. If $p_{a1} = p_{a2}$, then they share the orders and must bid $p_{b1} = p_{b2} \geqq S^{-1}[D(p_{a1})]$; while there are many possible subgame NE in this case, only the minimum bid is compatible with the equilibrium path. Finally, if $p_{a1} > p_{a2}$, then merchant 1 loses and $p_{b1} < p_{b2}$ is optimal. (3) The uniqueness proof is deferred to the Appendix. Q.E.D.

Thus, we see that the Bertrand merchant game with forward contracts yields the Walrasian price as the unique SPNE outcome regardless of the relation of p_c and the sales revenue maximizing price $\hat{p}$ (or the elasticity of demand η_c).

Proposition 3 extends to games with any number of merchants greater than one. The proof mimics that given here. Hence, "two" is sufficient for perfect competition.[8]

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[8]It should go without saying that while two is sufficient for perfect competition is this abstract model, we are certainly not suggesting that two is sufficient in reality. Numerous "realistic" modifications (e.g. differentiation among the merchants) would affect the results.

This result is also robust to the allocation scheme in the event of equal prices as long as both stages are coordinated; i.e. in the event of equal ask prices if merchant 1 gets a fraction α of the orders, then in the event of equal bids he must get α of the supplies. Coordination is needed to avoid defaults induced by the allocation scheme.

The extension of these results to the cases of vertical and horizontal supply are also different from section 4. When supply is vertical at $S < D(0)$, the SPNE has $p_{a1} = p_c$ for both players, but now we have a continuum of SPNE associated with the multiplicity of second stage NE with both merchants bidding a common price between 0 and p_c. Essentially we have an indeterminancy in who gets the surplus - the merchants or the suppliers. When $S \geqq D(0)$, we are really in the case of horizontal supply, and Proposition 3 holds. Of course, if supply is horizontal at p', then $p_c = p'$.

6. DISCUSSION.

The notion of "market-making" merchants who buy from suppliers and sell to consumers has been modeled as a two-stage Bertrand game. Remarkably, with one exception,[9] the SPNE prices

are the Walrasian prices (when SPNE exists). This result holds for any number of merchants greater than one. Thus, we have a (albeit partial equilibrium) model in which the Walrasian price arises not from the benevolent actions of a fictitious auctioneer but from optimal price setting behavior of merchants.

These results stand in sharp contrast to the results for capacity-constrained Bertrand competition even with capacity choice. The crucial difference is that in this paper there is "winner-take-all" competition for the inputs, whereas in the other papers either the inputs are taken as fixed or each player has an independent source for capacity production [Kreps and Scheinkman (1983); Vives (1983)]. The lesson is that it is not safe to ignor the input side or simply assume unrestricted availability of inputs at a constant marginal cost. Indeed, the difference between the Cournot outcome (under the latter assumption) and the zero-profit outcome indicates an incentive for vertical integration.

It is surprising that without forward contracts, a SPNE does not exist when $n_c < 1$ and bidding ties are resolved by equal allocation. In contrast, when there is forward

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[9]The lone exception is the game without forward contracts when $n_c < 1$ and bidding ties are resolved randomly, in which case the SPNE outcome has non-Walrasian prices. An alternative characterization of the SPNE (when it exists) for the model without forward contracts and any n_c is "zero merchant profits".

contracting, a unique SPNE always exists and it is Walrasian. Furthermore, when SPNE does exist without forward contracts (i.e. when bidding ties are resolved randomly), the consumer faces a monopolist. These results suggest an empirical prediction. Specifically, we might find a greater prevalence of forward contracting when $\pi_c < 1$. Of course, there are many other factors that might enter such as the durability of the commodity and repetitions of the game.

While the paper focused on one-shot play of the two-stage game, since the SPNE outcomes are unique, the same strategies will constitute the unique SPNE of every finitely repeated version of the game, and the outcome will be Walrasian in every period (with the one exception noted).[10] A crucial assumption in this extension to repeated play is that inventories not accumulate from one period to the next. The effect of inventory accumulation is the proper subject of future research.

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[10]Of course, the Folk theorems [Benoit and Krishna (1985); Fudenberg and Maskin (1983)] tell us that collusive outcomes can be sustained as perfect NE of infinitely repeated games. However, I do not believe that we have sufficient understanding of this discontinutiy at infinity to warrant putting much credibility on those results for practical application.

APPENDIX

Lemma A. Let $x_i = x$ for all i, and let $ER(x)$ denote $ER_i(x,x)$. For the Bertrand game in ask prices under uniform rationing: $ER(x) \leq M(2x)/2$.

PROOF: Let $\hat{x} \equiv D(\hat{p})$. Symmetry allows us to drop the subscript i. (a) For $x \leq \hat{x}/2$, $p_m = D^{-1}(2x)$ is the unique pure-strategy NE. This follows from Kreps and Scheinkman, Lemmas 2 and 3, since $\hat{x}$ is less than the Cournot quantity. Noting that $M(2x) = 2xp_m$ for $x \leq \hat{x}/2$, we have that $ER(x) = M(2x)/2$. (b) For $x > \hat{x}/2$, again from KS, Proposition 1, $ER(x) = \max\{p[D(p)-x]\}$, which defines the highest price, $p(x)$, in the support of the mixed-strategy. The first-order condition for the maximum is that $[D(p) - x + pD'(p)] = 0$. Differen tiating $ER(x)$ and using the F.O.C. gives $ER'(x) = - p(x) < 0$. Hence, $ER(x) < ER(\hat{x}/2) = \hat{p}\hat{x}/2 = M(2x)/2$ for $x > \hat{x}/2$. Q.E.D.

Extending Propositions 1 and 2 to N > 2.

The key to Lemma 1 was Lemma A above. One can show analogously that for any $N \geq 2$, $ER(x) \leq M(Nx)/N$. Then $ER(x) - xS^{-1}(Nx) \leq [M(Nx) - NxS^{-1}(Nx)]/N$, so the overbidding argument goes through. The rest is straightforward.

<u>Proof of Uniqueness Part of Proposition 3</u>.

(1) First we show there can be no other pure-strategy SPNE. Suppose contrariwise that (p_{a1}, p_{a2}) is part of a pure-strategy SPNE, and w.l.o.g. that $p_{a1} \geq p_{a2}$. If $p_{a2} < p_c$, then $E\pi_2 < 0$, which is clearly not optimal; hence, $p_{a2} \geq p_c$. If $p_{a2} > p_c$, then $E\pi_1 = 0$ if $p_{a1} > p_{a2}$, and $E\pi_1 = [p_{a2} - g^{-1}(p_{a2})]D(p_{a2})/2 > 0$ if $p_{a1} = p_{a2}$. But if merchant 1 undercuts p_{a2} ever so slightly, he can nearly double $E\pi_1$; hence, $p_{a1} \geq p_{a2} > p_c$ cannot be a NE. Since it has already been shown that p_c is the best response to $p_{a2} = p_c$, uniqueness among pure-strategies has been established.

(2) Let F_1 be a mixed-strategy probability distribution. The preceeding argument can be used to show that there can be no matched atoms at prices other than p_c. Then

$$E\pi_1(F_1,F_2) = \int \{[p_{a1} - g^{-1}(p_{a1})]D(p_{a1})[1-F_2(p_{a1})]\}dF_1 \ .$$

Note that atoms at p_c yield zero expected profits and so do not affect the above equation. The term in braces {} is negative for $p_{a1} < p_c$, so not surprisingly the lower bound of the support of F_1 is at least p_c. Moreover, for F_1 to be a best response to F_2, the term in {} must be constant on supp F_1, implying that

$$F_2(p_{a1}) = 1 - \pi_1/\{[p_{a1} - g^{-1}(p_{a1})]D(p_{a1})\} \quad ,$$

where π_1 is the constant expected profits. Notice that $F_2 < 1$

for all permissible p_{a1}, provided $\pi_1 > 0$; hence, there must be an atom at the upper bound of supp F_2, contradicting the preceeding argument. Therefore, $\pi_1 = 0$, which implies that F_2 is degenerate with all mass at p_c.

(3) Having established that the σ_1 family of strategies coincides with the set of SPNE strategies, we want to show that they all yield the Walrasian outcome. On the equilibrium path, $p_{a1} = p_c$, and both must bid at least $S^{-1}[D(p_c)] = p_c$. Clearly, if $p_{b1} > p_c$, then profits are negative, so on the equilibrium path $p_{b1} = p_c$ for all i. Q.E.D.

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